Peace Education in Math Classroom

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abstract

Probabilistic consideration shows that the chances of being aggressed and of being an aggressor are mathematically equal. This simple "theorem" may help people to assess its own army as a potential invader as well as an alleged "defence" force.

Armed forces are kept by alleging, without exception, that its purpose is to "defend" its own country and never for invasion to another country. If one argues for the abolition of force, he will be challenged by the question like "What if another country invades us?" This is also the case for the advocates of the Article 9 of the Japanese Constitution. Although this is a natural question and has to be answered anyhow, one has to remind that the following question should *equally* be posed: "What if our army invades another countries?"

However, we hardly hear this second question, not only in Japan. This may be based on our misconception that the former case is mathematically more probable, because the number of potential invader to his/her country is numerous but his/her country which may invade is single. But this is simply false. So it may be essential to 'reset' this popular fallacy and to remind ourselves that these possibilities are mathematically equal.

The following is an example of "theory" which may serve as a remedy for this misconception. It is not a realistic discussion but only a simple algebra indicating the starting point for, if you want, more sophisticated probabilistic theories. Nevertheless I believe it will help one to remind that these two questions, namely about the risk of being invaded and of becoming an invader, should be asked equally, equally frequently and equally seriously.

Suppose we have $n+1$ states whose probabilities of invasion to another states are all equal. Let p be this probability of invasion by *some* state to *any* other states in a certain period of time T. The quantity p is also an expected value of the *frequency* of invasion by a state in this period. Then the expected frequency of invasion by a state to *one* of other states is p/n. We assume no correlation among the behavior of different countries, e.g. alliance, exists.

Let us calculate the expected frequency of invasion to *some* state by *any* other states. The probability $P(k)$ of being invaded *simultaneously* during the period T by k states is

$$P(k) = \left(\frac{p}{n}\right)^{k}\left(1-\frac{p}{n}\right)^{n-k}$$

and its number of cases is ${}_{n}\mathrm{C}_{k}$.

Then the total expectation of *frequency* of invasion by *all* of other states is obtained by summing up all possible values of k weighted by ${}_{n}\mathrm{C}_{k}P(k)$:

$$\langle f\rangle = \sum_{k=1}^{n} k \ {}_{n}\mathrm{C}_{k} \ P(k) = \sum_{k=1}^{n} k \ {}_{n}\mathrm{C}_{k} \ \left(\frac{p}{n}\right)^{k}\left(1-\frac{p}{n}\right)^{n-k}$$

$$= \sum_{k=1}^{n} \frac{n\,!}{(n-k)\,!\ (k-1)\,!} \ \left(\frac{p}{n}\right)^{k}\left(1-\frac{p}{n}\right)^{n-k}$$

Replacing k with $r+1$, this expression becomes

$$\langle f\rangle = \sum_{r=0}^{n-1} \frac{n\,!}{(n-r-1)\,!\ (r)\,!}\left(\frac{p}{n}\right)^{r+1}\left(1-\frac{p}{n}\right)^{n-r-1}$$

$$= n\cdot\frac{p}{n} \ \sum_{r=0}^{n-1} \frac{(n-1)\,!}{(n-r-1)\,!\ (r)\,!}\left(\frac{p}{n}\right)^{r}\left(1-\frac{p}{n}\right)^{n-1-r}$$

$$= n\cdot\frac{p}{n} \ \left(\frac{p}{n}+1-\frac{p}{n}\right)^{n-1} = p$$

Namely, $\langle f\rangle$ is simply equal to p.

This result is quite an anticipated one because "defensive war" and "aggression" are only the different names for the same event, the war. This simple mathematical inference could be used as a teaching stuff in high school or in college math classrooms to promote the idea for peace and disarmament.